UDC 551.509.313(100)

A NOTE ON GLOBAL FORECASTING WITH THE KURIHARA GRID

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ABSTRACT

Kurihara and Holloway's approach of applying the box integration method to the Kurihara grid has been followed for a global forecast model. After encountering problems with the original system, it was found that the truncation error associated with both the orientation of grid points around each Pole and the computation scheme must be carefully considered. A slight alteration was made in Kurihara's grid which alleviated the troubles and provided acceptable forecasts.

1. INTRODUCTION

A primitive equation barotropic model with a free surface is used for this study. The equations are written in flux-divergence form. They are

$$\frac{\partial}{\partial t}(hu)+\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}uuh+\frac{\partial}{\partial\phi}vuh\cos\phi\right]-\frac{uvh\tan\phi}{a}-fhv=-\frac{h}{a\cos\phi}\frac{\partial}{\partial\lambda}g_0h, \quad (1)$$

$$\frac{\partial}{\partial t}(hv)+\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}uvh+\frac{\partial}{\partial\phi}vvh\cos\phi\right]+\frac{uuh\tan\phi}{a}+fhu=-\frac{h}{a}\frac{\partial}{\partial\phi}g_0h, \quad (2)$$

and

$$\frac{\partial}{\partial t}(h)+\frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}uh+\frac{\partial}{\partial\phi}vh\cos\phi\right]=0 \quad (3)$$

where a=mean radius of the spherical earth, ϕ=latitude, λ=longitude, h=height of the free surface, $u=a\cos\phi\dot{\lambda}$, $v=a\dot{\phi}$, f=Coriolis acceleration parameter, g_0=apparent gravity acceleration, and t=time. The energy-preserving box integration method proposed by Bryan (1966) with the D and L difference operators defined by Kurihara and Holloway (1967) is used to integrate the model. The time iteration scheme used is the Euler-backward method investigated by Kurihara (1965a). This iteration method sufficiently suppresses high-frequency oscillations to allow the use of analyzed wind and height fields, with no initialization, as initial data.

Integration of the model is carried out on the Kurihara grid (Kurihara, 1965b), which covers the earth with an approximately evenly spaced array of grid points. This system (called the regular grid in this paper) has a grid point at each Pole and $4\times(J-1)$ grid points spaced evenly around each of J equally spaced latitude circles between each Pole and the Equator. A polar stereographic view of the regular grid is in the top left of figure 1. A latitude increment of 3° is used, so there is one point at each Pole, four points at latitude 87.0°, eight points at latitude 84.0°, etc., to 120 points on the Equator. An alteration (called the 12-point grid in this paper) of the regular grid is produced by removing both Pole points, increasing the number of grid points in each row by eight, and moving each of these rows poleward by 1.5°. A polar stereographic view of the 12-point grid is also pictured in figure 1. Retaining the 3°-latitude increment, this grid has no Pole point, 12 points at latitude 88.5°, 16 points at latitude 85.5°, etc., to 128 points at latitude 1.5°. (There is no row of grid points on the Equator.) The increased resolution of the 12-point grid necessitates a reduction in the time step to 300 sec from the 900-sec time step of the regular grid. Graphs of an area of a box in a row vs.

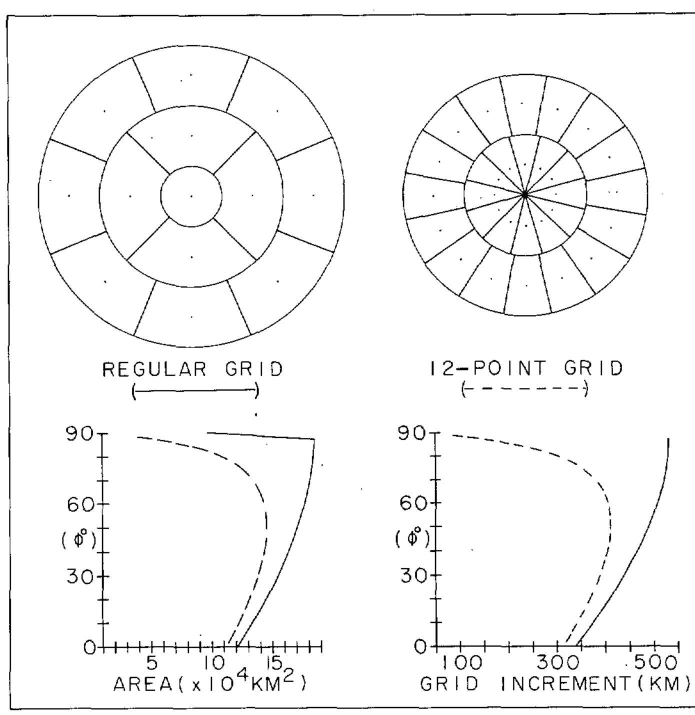


FIGURE 1.—Polar stereographic view of two Kurihara-type grids and other characteristics.

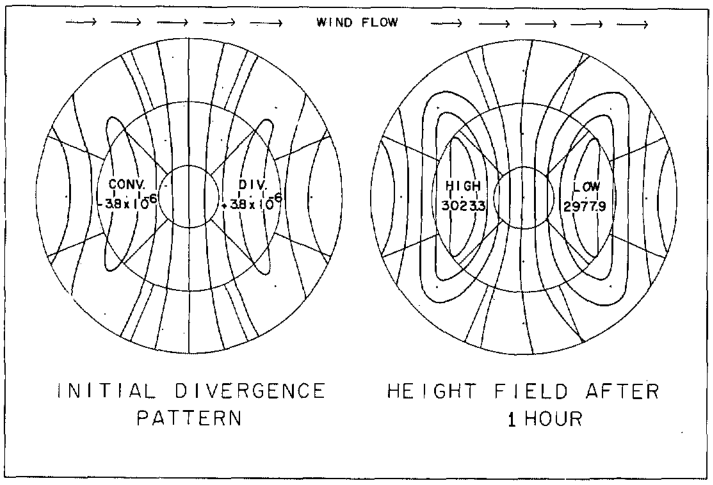


FIGURE 2.—Divergence and height fields for the cross-polar flow case.

latitude and longitudinal grid increment vs. latitude for both grids are in the bottom of figure 1.

2. THE TRUNCATION ERROR PROBLEM

In an unpublished paper, Shuman (1967) indicated the possibility of large truncation error at high latitudes on a grid which preserves longitudinal mesh length. To test the model for the problem, an analytic field (called the cross-polar flow in this paper) was used. The field is solid-body rotation around the earth and is a steady-state solution to the differential equations (1)–(3) for a nonrotating earth. The cross-polar flow is described by the equations:

$$u=-u_0 \sin \lambda \sin \phi, \tag{4}$$

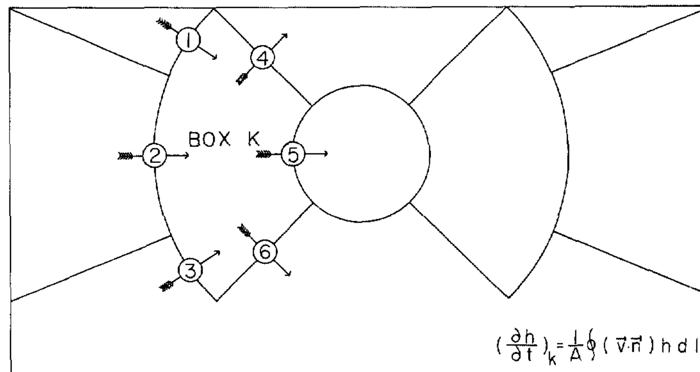
$$v=-u_0 \cos \lambda, \tag{5}$$

and

$$h=h_0-\frac{u_0^2}{2g_0} \sin^2 \lambda \cos^2 \phi \tag{6}$$

where u_0 and h_0 are constants of wind and height, and all other symbols are as previously described. Since the field is analytical nondivergent, the initial divergence pattern computed by the model will be a measure of the accuracy of the difference approximations and the grid. The cross-polar flow is a particularly good test of the truncation error at high latitudes, because its strongest flow goes directly across the Pole.

A regular grid forecast was made using this flow as initial conditions with $u_0=5$ m sec^{-1} and $h_0=3000$ m. Figure 2 illustrates the initial divergence pattern calculated by the model. An area of convergence is upwind of the Pole, and an area of divergence is downwind of the Pole, both of the order of 4×10^{-6} sec^{-1}, indicating that when flow across the Pole exists, there will be a spurious accumulation of mass upwind and a loss downwind of the Pole. This, in turn, will cause the development in the height



POINT	WEIGHT	INITIAL WIND	INITIAL HEIGHT	INITIAL FLUX
1	$+1.124\times10^{-6}$	+4.268 (V)	2999.9	$+1.439\times10^{-2}$
2	$+2.247\times10^{-6}$	+5.000 (V)	3000.0	$+3.371\times10^{-2}$
3	$+1.124\times10^{-6}$	+4.268 (V)	2999.9	$+1.439\times10^{-2}$
4	$+1.910\times10^{-6}$	−2.497 (U)	2999.9	-1.430×10^{-2}
5	$+1.410\times10^{-6}$	−5.000 (V)	3000.0	-2.249×10^{-2}
6	$+1.910\times10^{-6}$	−2.497 (U)	2999.9	-1.430×10^{-2}
			NET FLUX IN BOX K	$+1.139\times10^{-2}$

FIGURE 3.—An example of the computation of divergence for the cross-polar flow case.

field of a high center upwind of the Pole and a low downwind. The height field after 1 hr is shown in figure 2. The high and low centers are in the expected positions, and both show departures of 20 m from the steady-state height field. Since the height difference in the steady-state height field between the Pole and the Equator is only a few meters, the deviations are of considerable magnitude. After 1 hr, the deviations in the forecast height field continuously change orientation, but do not grow. Since the deviations are largest at high latitudes, the computation of divergence in the first box upwind of the Pole was investigated to discover the reason for their existence. The computation is illustrated in figure 3.

3. DIAGNOSIS OF THE TRUNCATION ERROR SOURCE

To apply the box integration method to the system, each of the Kurihara grid points is surrounded by an element of surface area, or a box. For an arbitrary box, the divergence of the mass field may be written as

$$\frac{\partial}{\partial t}(h)=\frac{1}{A}\oint(\mathbf{v}\cdot\mathbf{n})h\,dl \tag{7}$$

where $\mathbf{v}$ is the velocity vector on the boundary of the box, $\mathbf{n}$ is the outward-directed unit vector normal to the boundary of the box, l is the boundary of the box, A is the area of the box, and h is the height of the free surface. To calculate the divergence for box K in figure 3, the product of the wind ($\mathbf{v}$), height (h), and weight

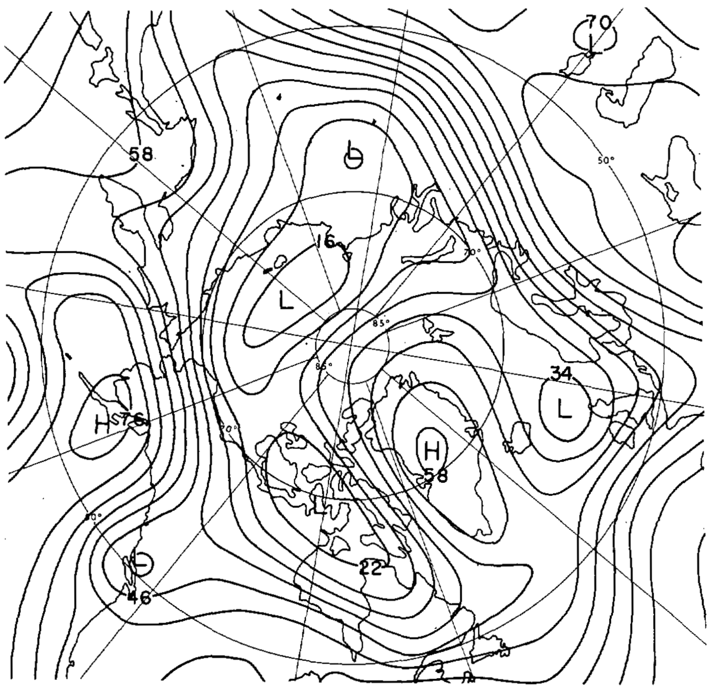


FIGURE 4.—Initial analysis height field, 500 mb at 12 GMT on May 12, 1968 (contour interval 60 m).

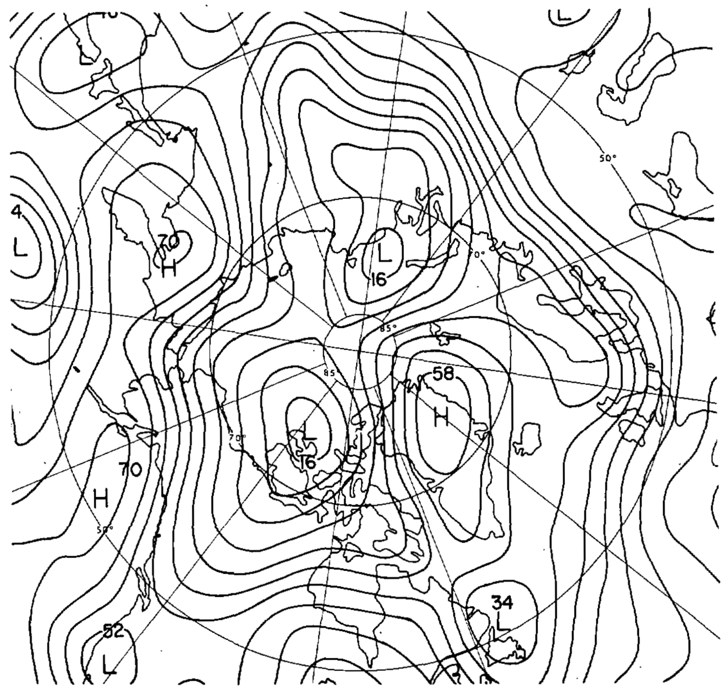


FIGURE 5.—Verification analysis height field, 500 mb at 00 GMT on May 14, 1968 (contour interval 60 m).

$\left(\dfrac{\text{length of line segment}}{\text{area of box}}\right)$ is computed at each of line segments (1)–(6), and the six products are summed up. The values of wind and height on the boundary are obtained by averaging the values in the two boxes adjacent to each line segment. In figure 3, a positive product indicates an inflow of mass, and a negative product indicates an outflow of mass. In box K, there is inflow at segments (1), (2), (3) and outflow at segments (4), (5), and (6). The result is a net inflow, or convergence, of mass.

To calculate the winds at segments (2) and (5), the values of wind resolved on the axes of two parallel coordinates are averaged. However, to obtain the values of wind at segments (1), (3), (4), and (6), the values of wind resolved on nonparallel coordinate axes are averaged. This distinction is largest for segments (4) and (6), where in both cases, the two sets of coordinate axes have been rotated through 90° of longitude. The weights, which are large for segments (4) and (6), magnify the problem and lead to a net underestimation of the outflow from box K. The situation is reversed for the box opposite box K. It is important to note that the problem of spurious anticyclogenesis will not appear when the flow is completely zonal. Such a flow does not adequately test the grid.

4. TESTS WITH REAL DATA

For the first test of the importance of the error in a forecast from real data, an analysis was chosen that featured significant flow across the Pole. The Northern Hemisphere flow pattern for 12 GMT on May 12, 1968, is pictured in figure 4.

The features of prime importance on this analysis affected by spurious divergences are the strong flow across the Pole and the large blocking High over Greenland. In the verification analysis (fig. 5) for 36 hr later, 00 GMT on May 14, 1968, the cross-polar flow has receded to just off the Pole, and the High has maintained both its position over Greenland and its central height of 5580 m.

The first forecast of this case was computed with the regular grid. The 36-hr height field forecast is shown in figure 6. Drastic and unacceptable changes are predicted in the polar region. The cross-polar flow has been completely eliminated and replaced by a strong High, and the High formerly over Greenland has disappeared. It was to avoid this spurious anticyclogenesis that the regular grid was altered. The alteration, the 12-point grid, was described earlier. The 36-hr height field forecast for this case by the 12-point grid is shown in figure 7. Note the large difference between the 12-point grid and the regular grid forecasts in the polar area. The slight alteration in the grid provides a forecast that maintains both the cross-polar flow and the High over Greenland, although the latter is moved somewhat north and its central height is increased by 80 m.

For a second test of the problem of spurious high-latitude anticyclogenesis, a case with almost no flow across the Pole was chosen. The initial analysis for this case is at 12 GMT on Mar. 27, 1968, and is pictured in figure 8.

There is general low pressure in the polar region. The low center nearest the Pole is of particular interest.

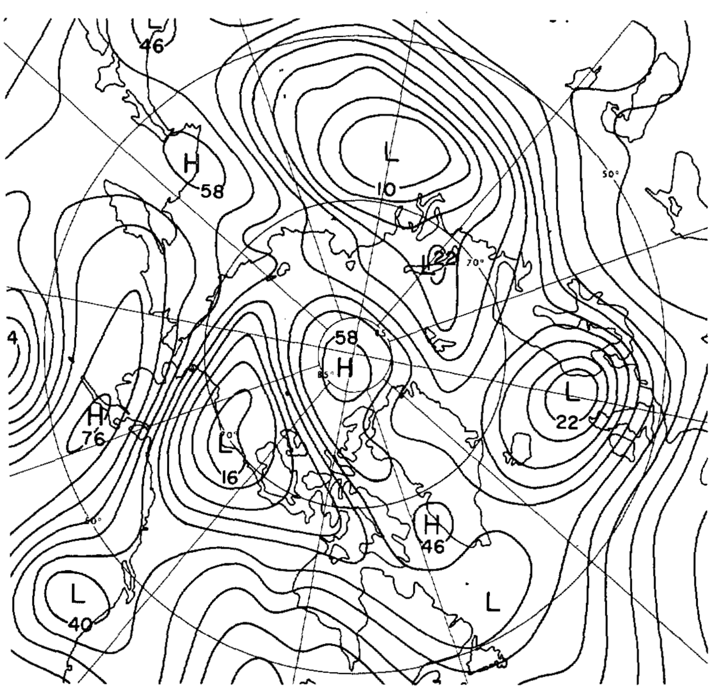


FIGURE 6.—Thirty-six-hour regular grid height field forecast from 12 GMT on May 12, 1968, 500 mb (contour interval 60 m).

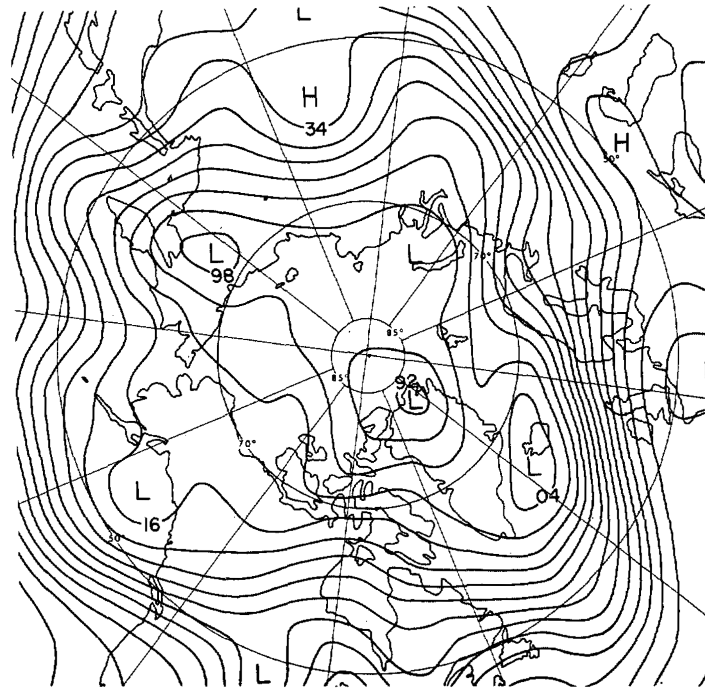


FIGURE 8.—Initial analysis height field, 500 mb at 12 GMT on Mar. 27, 1968 (contour interval 60 m).

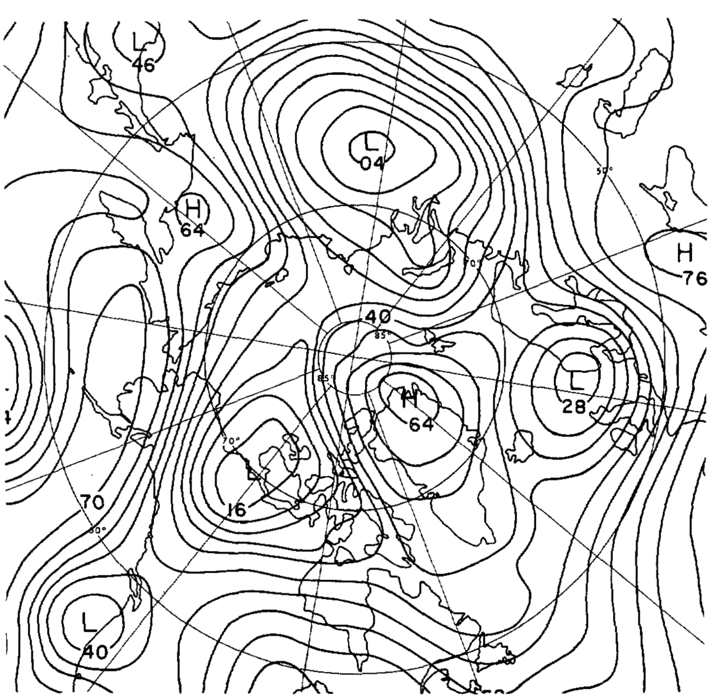


FIGURE 7.—Thirty-six-hour 12-point grid height field forecast from 12 GMT on May 12, 1968, 500 mb (contour interval 60 m).

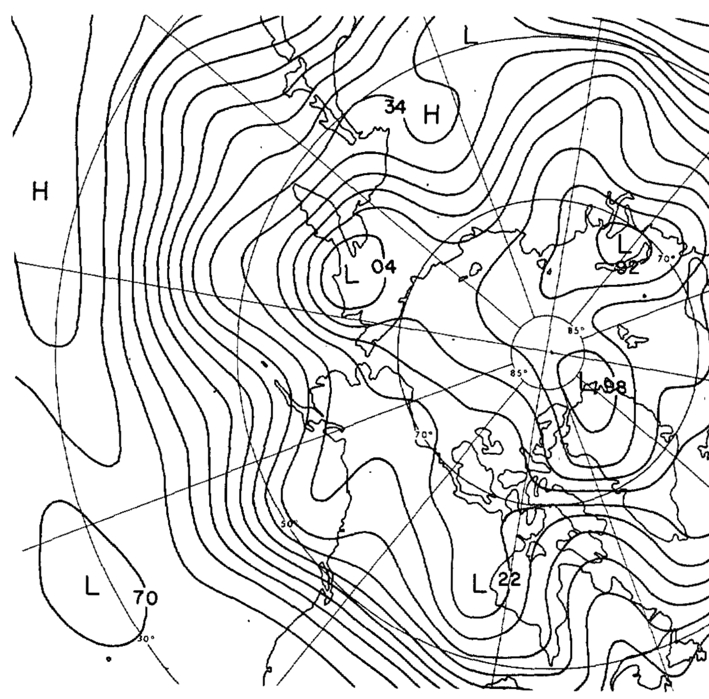


FIGURE 9.—Verification analysis height field, 500 mb at 00 GMT on Mar. 29, 1968 (contour interval 60 m).

Initially, it is over northern Greenland with a central height slightly below 4920 m. In the verification analysis 36 hr later, figure 9, this feature has remained stationary, and its central height has risen somewhat. The 36-hr regular grid forecast for this case is shown in figure 10. Not surprisingly, the spurious polar anticyclogenesis is not forecast with this flow pattern. The polar heights have risen slightly, and the Greenland Low has been filled a bit,

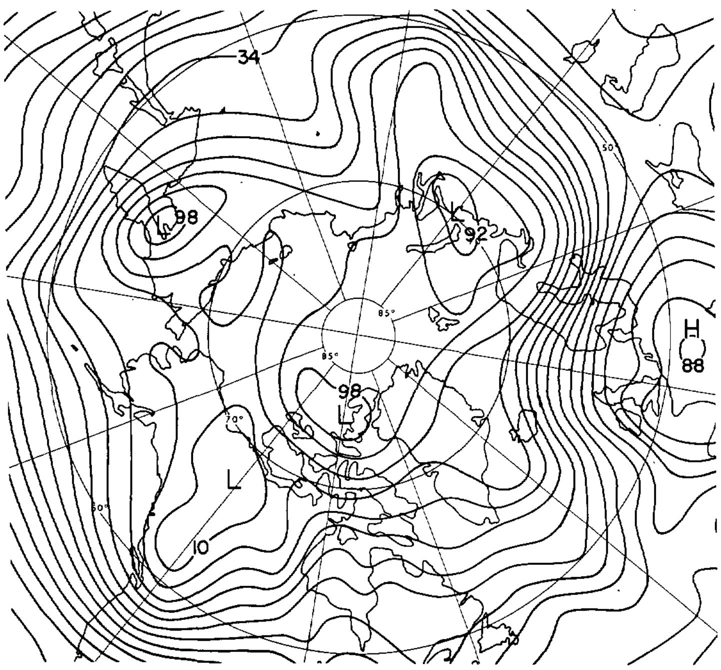


FIGURE 10.—Thirty-six-hour regular grid height field forecast from 12 GMT on Mar. 27, 1968, 500 mb (contour interval 60 m).

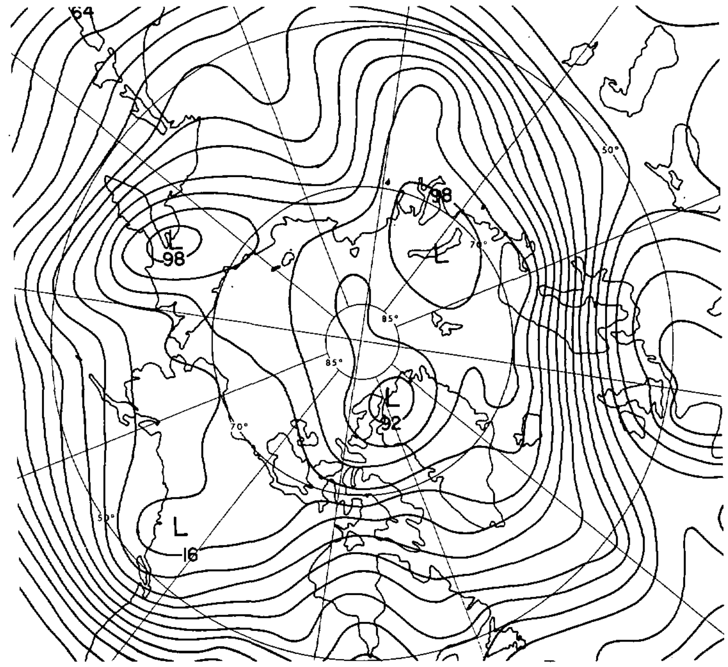


FIGURE 11.—Thirty-six-hour 12-point grid height field forecast from 12 GMT on Mar. 27, 1968, 500 mb (contour interval 60 m).

both correctly, although the Low has retrograded 60° longitude. The problem of phase speed in a forecast with a Kurihara type of grid, with low resolution at high latitudes, has been mentioned by Gates and Riegel (1963) and by Grimmer and Shaw (1967). We have not done a thorough analysis of this problem for the model; however, it is noteworthy that the 36-hr 12-point grid forecast for this case, in figure 11, apparently corrected much of the retrogression error by reducing it to less than 15° longitude. The heights at the Pole and the low center have correctly been raised slightly, both similar to the regular grid forecast.

5. SUMMARY

The approach of Kurihara and Holloway (1967) has been taken, and Kurihara's original grid along with the computation scheme has been found to produce spurious polar anticyclogenesis under conditions of cross-polar flow. The original grid was altered to provide more resolution at high latitudes. The alteration reduced this problem to acceptable levels for a primitive equation barotropic model.

ACKNOWLEDGMENTS

Many thanks are due Dr. J. Gerrity of the National Meteorological Center with whom I have worked and Dr. Y. Kurihara and J. L. Holloway of the Geophysical Fluid Dynamics Laboratory, who provided valuable guidance in setting up the model and participated in several valuable discussions of the results.

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[*Recieved January 9, 1969; revised March 3, 1969*]